Measuring Skewness Premia in the Cross-section of Hedge Fund Returns

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Abstract

This paper empirically investigates the respective ability of systematic and idiosyncratic skewness in explaining the cross-sectional differences in hedge fund returns. We find that idiosyncratic skewness is a significant factor in explaining and predicting the cross-section of hedge funds returns, even when controlling for other types of risks and funds characteristics. Systematic skewness is a significant factor only when we exploit a machine learning approach to recover missing data of funds returns. Hedge funds sorted by idiosyncratic skewness outperform those in the bottom quintile by 0.27% per month on a risk-adjusted basis. However, skilled hedge funds significantly price systematic skewness while exploiting idiosyncratic skewness to generate alpha. Both systematic and idiosyncratic skewness are significantly positively associated with market timing, but such an activity is dominated by alpha-generating activities in the cross-section of funds returns.

Keywords: Hedge Fund Performance, Systematic Skewness, Idiosyncratic Skewness, Matrix Completion, Missing Data

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1. Introduction

This study revisits the ability of skewness to explain the cross-sectional dispersion of hedge fund returns. Previous studies have shown that sophisticated strategies executed by hedge funds cause significant tail risk exposure, which might not be diversifiable (see among others Fung and Hsieh, 1997, 2001; Mitchel and Pulvino, 2001; Agarwal and Naik, 2004; Fung et al., 2008; Brown et al., 2012). Based on that evidence, Bali et al. (2012) assess the power of aggregate risk measures, including skewness, to explain hedge fund returns, and they find only a significant relationship between variance and expected fund returns and not for higher moments. However, studying the theoretical link between skewness and asset prices, previous research has proposed models which decompose skewness in its systematic and idiosyncratic components as important factors in pricing securities (see Brunnermeier et al., 2007; Barberis and Huang, 2008; Mitton and Vorkink, 2007; Dahlquist et al., 2017; Langlois, 2020). For that reason, we believe that it is worth empirically investigating the corresponding importance of systematic skewness (or coskewness) and idiosyncratic skewness in explaining the cross-section of hedge fund performance.

We find that both univariate and bivariate portfolio-level analyses and the cross-sectional regressions reveal a positive and significant relationship between idiosyncratic skewness and expected returns of 10,516 hedge funds over 1994-2018. Impressively, the average return difference between high and low quintile portfolios is 3.2% per annum with a Newey and West *t*-statistic of 4.06, while the *t*-statistic of the relevant coefficient of cross-sectional regressions ranges from 4.39 to 4.75. However, this is not always the case for coskewness. The predictive power of idiosyncratic skewness on funds' returns remains when we control for several funds' characteristics in our cross-sectional regressions. More importantly, the above significant link persists even when we include total variance and its decomposition in systematic and unsystematic risk as control variables in our cross-sectional regressions. Bali et al. (2012) have found that total variance and systematic risk are strong predictors of hedge fund returns. On the other hand, our findings show that both volatility measures are insignificant when considering idiosyncratic skewness, making it a more powerful predictor of hedge fund returns.

Interestingly, we find that high skilled funds price also coskewness apart from idiosyncratic skewness, which is not the case for low skilled hedge funds. In particular, a portfolio analysis indicates a negative and significant link between coskewness and expected returns of high skilled hedge funds. The average return difference between high and low quintile portfolios is -2.26% per annum with a Newey and West *t*-statistic of -1.97. These findings align with the theory of coskewness risk premium (e.g., Kraus and Litzenberger, 1976). We also verify that hedge funds are good market timers because coskewness and idiosyncratic skewness estimates based on time-varying betas on public information are significant and positively related to expected returns. However, their successful market timing is dominated by alpha-generating activities of hedge funds. This is because we observe a strong negative link between coskewness based on *non*-time-varying betas and funds returns and a strong positive correlation between the relevant idiosyncratic skewness estimates and funds returns. The corresponding *t*-statistics of the relevant regression coefficients are -3.15 and 4.65, respectively.

The corresponding empirical findings of recovered hedge fund data based on the *matrix completion* technique verify the positive and significant relationship between idiosyncratic skewness and expected fund returns. But more importantly, they unveil a negative and significant relationship between systematic skewness and hedge fund returns, consistent with the coskewness pricing theory of Kraus and Litzenberger (1976). The hedge fund coskewness risk premium varies from -0.72% to -0.11% per annum, with the corresponding Newey and West (1987) *t*-statistics ranging from -1.97 to -2.70. The univariate portfolio-level analysis also shows that the average return difference between high and low quintile coskewness portfolios is -2.16% per annum with a Newey and West *t*-statistic of -2.48.

Our work is related to different strands of literature. Many studies highlight the link between return skewness and asset prices, especially in equities markets (see Brunnermeier et al., 2007; Boyer et al., 2010; Amaya et al., 2015). Investors with fully diversified portfolios will demand compensation for bearing negative coskewness with the market portfolio (i.e., systematic skewness), which should also explain the cross-sectional dispersion of expected returns across firms (see Kraus and Litzenberger, 1976; Harvey and Siddique, 2000; Dittmar 2002; Chabi-Yo, et al., 2014). On the other hand, other studies show idiosyncratic skewness' ability to predict future returns because of investors' *gambling* preferences (Mitton and Vorkink, 2007; Barberis and Huang, 2008; Kumar, 2009; Boyer et al., 2010). For instance, investors with skewness preferences hold under-diversified portfolios and invest more in positively skewed securities, and such behavior results in stocks with idiosyncratic skewness paying a premium. Our paper is generally connected to the studies on the importance of return skewness for portfolio choice. Dahlquist et al. (2017) show that skewness can explain empirical asset allocation choices in a generalized disappointment aversion framework, and Ghysels et al. (2016) empirically report this feature of skewness across emerging markets. As far as we know, no paper has investigated the explanatory and predictive power of coskewness, and idiosyncratic skewness on the cross-section of hedge fund returns simultaneously.

Our work is also related to papers investigating the risk-return characteristics of hedge fund returns. Those mainly include studies focusing on fundamental risks (e.g., macroeconomic and liquidity risk).[2] Bali et al. (2012) concentrate on aggregate risks related to the first moments of

[2] A partial list includes Fung and Hsieh (1997, 2004), Ackerman et al. (1999), Agarwal and Naik (2004), Kosowski et al. (2007), Fung et al. (2008), Agarwal et al. (2009), Jagannathan et al. (2010), Sadka (2010), Titman and Tiu (2011), Bali et al., (2011, 2012, 2014), Chen et al., (2017).

hedge fund returns and their explanatory power on the cross-sectional dispersion of funds returns. They find that only total variance and systematic risk have predictive power on the cross-section of hedge fund returns, while residual risk, total skewness, and kurtosis do not. Heuson et al. (2019) recently also utilize idiosyncratic skewness in hedge fund performance evaluation. They also find that idiosyncratic skewness, computed as the skewness of the residuals from the Fung and Hsieh (2004) factor model, can predict hedge fund returns. One of the essential differences between their paper and ours is that they study only idiosyncratic skewness, not coskewness risk premium of hedge fund returns. In addition, we use skewness estimation methods suitable for hedge funds, which consider that funds trade in different horizons and outliers of funds, returns that can generate biased skewness estimators. We also examine the critical determinants of the above skewness risk premiums. The above studies explain hedge fund returns based on their exposure to fundamental risks such as macroeconomic factors or first moments. Recently, Chen et al. (2021) show that non-fundamental shocks, such as sentiment fluctuations, can also explain hedge fund returns. They find a statistically and economically significant relationship between a sentiment beta subsequent fund returns in the cross-section.

Our study is also linked with the recent literature on machine learning and big data applications in empirical asset pricing. Gu et al., (2020) exploit the power of numerous machine learning methods to forecast stock returns based on an extensive set of predictors in large-scale comparative analysis. Feng et al. (2020) a *LASSO two-pass regression* specification to assess the contribution to asset pricing of a new factor within a large universe of existing factors. Giglio et al., (2021) revisit hedge funds' skill to generate alphas while controlling for data snooping effects based on machine learning approaches for multiple hypothesis testing and *matrix completion*.

We make several contributions to the existing literature. First, we use up-to-date and suitable methods for measuring coskewness and idiosyncratic skewness of hedge funds. We adopt a coskewness estimation method suitable for the empirical cross-sectional dispersion of funds as Back et al. (2018) measure for mutual fund returns. They define coskewness as the covariance of the payoff of a zero-cost portfolio and the squared market excess return. Like mutual funds, hedged funds operate over different periods, so existing coskewness measures, such as the one given by the quadratic regression, do not generate proportional empirical estimates for each fund. We also use a quantile-based measure of idiosyncratic skewness for hedge fund returns like Ghysels et al. (2016) for emerging markets, which is robust to the presence of outliers and does not involve the use of options data and it can be computed at various horizons. The last feature is essential for the case of hedge funds operating in shorter-horizons, whose returns might be more sensitive to significant outliers. We capture the standardized difference of the distance between the top percentile and the median and the distance between the median and the bottom percentile of the idiosyncratic skewness estimated from the monthly time series residuals of a three-moment CAPM applied on hedge funds returns. In each month for each fund, we calculate both measures of skewness based on their information of the prior 36 months. Second, we use the above coskewness and idiosyncratic skewness estimates in formal asset pricing tests. As main exercises, we conduct portfolio analyses by constructing five equally-weighted portfolios based on the skewness estimates. We also form double-sorted portfolios based on each skewness estimate while controlling for the other. Apart from the raw returns of each portfolio and the spread portfolio of their high and low quintile, we assess the risk-adjusted returns of a global Carhart (1997), Fung and Hsieh (2004), and a six-factor model similar to Della Corte et al. (2020). To assess the predictive ability of coskewness and idiosyncratic on hedge fund returns skewness, we run Fama and MacBeth (1973) cross-sectional regressions of our one-month-ahead funds' returns on our skewness estimates while controlling for numerous hedge fund characteristics. We also run similar cross-sectional regressions while controlling for significant predictors of hedge fund returns, such as total and systematic risk. Our third contribution examines the main drivers of skewness risk premia of hedge fund returns. As a first driver, we assess fund managers' skills using Titman and Tiu's (2011) approach to divide our funds' sample into high and low skilled funds based on R-squared estimation from Fung and Hsieh (2004) factor model. Then we perform a portfolio analysis as above for each class of funds separately. We evaluate market timing as the second driver of skewness risk premia of funds returns. We divide coskewness and idiosyncratic skewness into two components, one that is due to time-varying market exposures and another one that is due to other factors using Ferson and Schadt's (1996) regression model. We run cross-sectional regressions of each component on funds' future returns and for each skewness estimate.

Finally, we adopt the *matrix completion* approach from the recent machine learning literature (see, Candes and Tao, 2009; Mazumder et al., 2010; Koltchinskii et al., 2011) to recover missing entries in the matrix of hedge fund returns. The most popular application of this method is the *Netflix* ratings problem, in which the task is to predict viewers' ratings of no-rated movies based on the existing ones. Because only a fraction of viewers rates a small number of movies, leading to many missing entries of ratings. This method is strictly associated with hedge fund returns because their corresponding time series have short histories or missing entries of data, while they constitute high dimensionality matrices. Our *matrix completion* application uses the structure and observed data of funds returns to efficiently recover the missing elements in a low-rank matrix of hedge fund returns to estimate their systematic and idiosyncratic skewness.

The paper is organized as follows. Section 2 provides the theoretic framework of measuring coskewness and idiosyncratic skewness of hedge fund returns. Section 3 describes the sample data. Section 4 presents the main empirical findings of cross-sectional regressions and portfolio analyses. Section 5 examines the main drivers of the skewness-fund performance relationship. Section 6 present the methodology of *matrix completion* and the corresponding empirical findings. Section 7 concludes the paper.

2. Measuring Coskewness and Idiosyncratic Skewness

In this section, we discuss the asset pricing implications of coskewness. We start with the definition of coskewness and the inability of past measures to accurately capture coskewness, especially in hedge fund returns, in Section 2.1. We then describe in Section 2.2 the approach of Back et al. (2018) in measuring the coskewness of mutual funds, which considers the successful market timing of fund managers and could also be applicable to hedge fund returns.

2.1. Definition and Estimation of Hedge Fund Coskewness

Coskewness is the covariance of the excess return of an asset with the squared benchmark return (e.g., the market portfolio excess return) as the three-moment CAPM gives it.[3] Under such a framework, investors prefer higher portfolio return skewness. Hence, they add (discard) assets with positive (negative) coskewness to the market portfolio as this will make the latter more positively (negatively) skewed. If investors consider the first three moments of returns for their investment decisions, then there should be an exact negative relationship between the CAPM alpha and coskewness. In other words, investors require a higher alpha for holding funds that decrease the market portfolio's skewness as long as they have decreasing absolute risk aversion. Thus, the coskewness risk (i.e., systematic skewness) should be negative (see

[3] The three-moment CAPM is given as $E_{t-1}[r_{i,t} - r_{f,t}] = \gamma_{M,t} Cov_{t-1}(r_{i,t}, r_{M,t}) + \gamma_{M^2,t} Cov_{t-1}(r_{i,t}, r_{M,t}^2)$, where $r_{i,t}$ is the return of an asset i for period t; $r_{M,t}$ is the market portfolio return for period t; $r_{f,t}$ is the risk – free rate for period *t*; $\gamma_{M,t}$ and $\gamma_{M^2,t}$ are, respectively, the time *t* prices of covariance and coskewness risk; and $E_{t-1}[\cdot]$ and $Cov_{t-1}(\cdot)$ represent, respectively the expected return and covariance conditional on information at time $t-1$. So, coskewness is given by $Cov_{t-1}(r_{i,t}, r_{M,t}^2)$, while $\gamma_{M^2,t}$ represents the coskewness risk.

Kraus and Litzenberger, 1976). In our case, the trade-off between alpha and coskewness could also be translated as an alpha cost to seeking positive coskewness, due to fund managers main goal to "seek alphas" (see also Back et al., 2018). On the other hand, successful market timing of fund managers should be able to eliminate that cost and lead to a positive correlation between alpha and coskewness.

There are several approaches to estimate the three-moment CAPM, and so coskewness, in the literature (see among others, Rubinstein, 1973; Kraus and Litzenberger, 1976; Harvey and Siddique; 2000, Dittmar, 2002; Dahlquist et al., 2017). According to Frisch-Waugh Theorem, a commonly accepted measure of coskewness for an asset with return r_i is obtained by the quadratic regression, motivated by the beta representation of the three-moment CAPM,

$$r_{i,t} - r_{f,t} = a_i + b_{1,i}\left(r_{M,t} - r_{f,t}\right) + b_{2,i}\left(r_{M,t} - r_{f,t}\right)^2 + \epsilon_{i,t} \tag{1}$$

where $r_{i,t}$ is the return of an asset i for period *t*; r_f is the risk-free rate for period *t*; $r_{M,t}$ is the market portfolio return for period; a_i is the intercept; $\epsilon_{i,t}$ are the quadratic regression residuals for period *t*; $b_{1,i}$ and $b_{2,i}$ represent the systematic risk and systematic skewness, respectively. [4] The systematic skewness term, $b_{2,i}$ is referred to as coskewness across the relevant literature. Treynor and Mazuy (1966) point out that a positive coskewness value (i.e., $b_{2,i} > 0$) is indicative of successful market timing ability, while a negative value reflects incorrect market timing ability by fund managers.

Market timing literature has raised the weakness of $b_{2,i}$ from equation (1) to accurately measuring market timing. They may have raised that weakness in various aspects, but not precisely in the context of coskewness pricing model and its relation to $b_{2,i}$. For example, the

[4] Another closely related measure of coskewness is the standardized measure of Harvey and Siddique (2000), whose cross-sectional results are most of the time qualitatively equal with that of quadratic regression estimation of coskewness.

issue has been found due to time-varying betas (Edelen 1999; Ferson and Schadt, 1996; Ferson and Warther, 1996) because of options' or assets' with option-like payoffs usage (Jagannathan and Korajczyk, 1986). Recently, Back et al. (2018) have emphasized that the quadratic term coefficient (i.e., $b_{2,i}$) is a weak measure of coskewness in the empirical cross-section of mutual fund returns. In that case, the deviation of $b_{2,i}$ from actual coskewness holds at least quantitatively. The market moments relating $b_{2,i}$ to actual coskewness, as defined above, will be different for different funds since mutual funds operate over different time horizons.

As hedge funds run their operations in different periods and our dataset includes both live and graveyard funds, it is important to focus on a coskewness estimation method appropriate for fund returns, which also accounts for the successful market timing of hedge fund managers. Hence, we follow Back et al. (2018) and apply their coskewness estimation approach on hedge funds returns. Applying a general CAPM framework of the hedge funds excess return on the market excess return we take

$$r_i - r_f = \alpha_i + \beta_i(r_M - r_f) + \varepsilon_i \tag{2}$$

where r_i is the hedge fund return; α_i is the intercept and hedge funds alpha; β_i is the beta coefficient of the fund; ε_i are the CAPM regression residuals. The coskewness is then given by

$$cov\left(\alpha_i + \varepsilon_i, (r_M - r_f)^2\right) = E\left[\varepsilon_i(r_M - r_f)^2\right] \tag{3}$$

This is consistent with the general definition of coskewness, as described above, because the random variable $\alpha_i + \varepsilon_i$ represents the payoff of a zero-cost portfolio similar to the excess return of a hedge fund. The above equation shows that mean-variance-skewness investors should care for positive coskewness and positive alpha when investing in a hedge fund's return, and this is also strictly related to the successful market timing of hedge fund managers.

To estimate the degree of coskewness for each hedge fund at a given period t, we use a rolling window OLS regression of 36 monthly excess returns for each hedge fund i, as shown in (3).[5] We then estimate coskewness as

$$CS_{i,t} = \frac{1}{T_i} \sum_{t=1}^{T_i} \varepsilon_{i,t} \left(r_{M,t} - r_{f,t}\right)^2 \tag{4}$$

where T_i is the window sample size and $\varepsilon_{i,t}$ are the CAPM regression residuals in a month t over the corresponding window of observations $t - 36$ to t, which are orthogonal to the excess market return. We allow time variation in our parameters (see also Bali *et al*., 2012).

2.2. Definition and Estimation of Hedge Fund Idiosyncratic Skewness

As already discussed above, numerous studies have proved that mean-variance-skewness investors price the coskewness of an asset with the market portfolio (Kraus and Litzenberger, 1976; Harvey and Siddique; 2000, Dittmar, 2002). However, coskewness is not always the only channel that investors' preference for skewness explains cross-sectional returns. For example, investors with skewness preference hold under-diversified portfolios due to taking advantage of the upside potential of positively skewed assets (see Simkowitz and Beedles, 1978; Conine and Tamarkin, 1981; Brunnermeier et al., 2007; Mitton and Vorkink, 2007). Such an investor's behavior could be attributed to managerial skill or lottery preferences (i.e., gambling behavior). If the former holds, then higher idiosyncratic skewness should be associated with the higher future performance of hedge funds. On the other hand, Mitton and Vorkink, (2007) reveal that both coskewness and idiosyncratic skewness are priced from investors. As a result, assets with higher idiosyncratic skewness should earn lower returns or, in other words, require a negative risk premium (see also, Brunnermeier et al., 2007; Barberis and Huang, 2008).

[5] We have also employed a rolling window OLS regression of 60 monthly excess returns with similar findings.

Idiosyncratic skewness is defined as the skewness of the residuals from a regression model. In general, Bali et al. (2016) propose the Fama and French (1993) three-factor model regression, Jondeau et al. (2019) and Langlois (2020) use the three-moment CAPM regression to extract the residuals for idiosyncratic skewness calculation. Since our study is closer to those of Jondeau et al. (2019) and Langlois (2020), there is no exact approach for idiosyncratic skewness calculation of hedge fund returns, and in order to be consistent with the coskewness estimation, as described above, we adopt the approach of quadratic regression.[6] Hence, using (1) to extract $\epsilon_{i,t}$ and the formula of skewness, the exact definition of idiosyncratic skewness can be given by

$$is_{i,t} = \frac{\frac{1}{T_i}\sum_{t=1}^{T_i} \epsilon_{i,t}{}^3}{\left(\frac{1}{T_i}\sum_{t=1}^{T_i} \epsilon_{i,t}{}^2\right)^{3/2}} \tag{5}$$

Measuring idiosyncratic skewness as above is, as a matter of fact, a difficult task. This is because raising residuals to the third power, as in equation (5), makes skewness estimation vulnerable to outliers (see also Neuberger, 2012; Ghysels *et al*., 2016; Jondeau *et al*., 2019). Hence, it is crucial to estimate asymmetry measures in a way that is robust to outliers by definition. Several studies have attempted to overcome this issue, with most of them focusing on measures based on option prices (Bakshi et al., 2003; Conrad et al., 2013; Del Viva et al., 2017).[7]

Recent literature proposes measures, which should be calculated with respect to quantiles computed in the tail of the distribution of returns (Garcia et al., 2014; Ghysels et al., 2016). We follow Ghysels et al. (2016), who propose a method robust to outliers, not involving options

[6] We have also used Fung and Hsieh's (2004) factor model regression residuals, and our overall results remain the same.

[7] Other studies have focused on cross-sectional moments (Kapadia, 2012) or usage of high-frequency data (Neuberger, 2012; Amaya et al., 2015) to measure skewness accurately. However, those studies have been mainly concentrated on realized and not idiosyncratic skewness, contrary to our study.

data, and applicable at various horizons. Even though their study focuses on emerging equity markets returns, whose distribution presents a higher degree of asymmetry, economic and political conditions are highly likely to be associated with changes in the distribution of other asset returns such as hedge funds. Except for the fact that our dataset includes funds from emerging markets, it also includes funds operating in different horizons, as explained above. This means shorter-horizon funds' returns may be more sensitive to significant outliers, mainly when calculating idiosyncratic skewness.

Hence, likewise Ghysels et al. (2016) and Langlois (2020), we estimate a quantile-based measure of hedge funds for idiosyncratic skewness. This is the first time that such a measure has been used for hedge fund returns. Their approach measures the standardized difference between the distance of the highest percentile and the median and the distance between the median and the lowest percentile. Then in our case, the idiosyncratic skewness is given by using a rolling window quadratic regression of 36 monthly excess returns for each hedge fund i, as

$$IS_i = \frac{\left(q_{0.95}(\epsilon_{i,t}) - q_{0.50}(\epsilon_{i,t})\right) - \left(q_{0.50}(\epsilon_{i,t}) - q_{0.05}(\epsilon_{i,t})\right)}{q_{0.95}(\epsilon_{i,t}) - q_{0.05}(\epsilon_{i,t})} \tag{6}$$

where, $\epsilon_{i,t}$ are the quadratic regression residuals in a month t over the corresponding window of observations $t - 36$ to t; $q_{0.05}(\cdot)$, $q_{0.50}(\cdot)$ and $q_{0.95}(\cdot)$ are the 5th, 50th and 95th empirical percentiles of $\varepsilon_{i,t}$. IS_i is zero for a symmetric distribution and positive (negative) for positively (negatively) skewed distribution.

3. Data

We use Lipper TASS to obtain our hedge fund data set for our study. Lipper TASS contains information on monthly net-of-fee returns and monthly assets under management for hedge funds and CTAs and specific fund characteristics, such as management and incentive fees charged to investors. From January 1994 to December 2018, Lipper TASS contains the relevant information for 35,873 live and "graveyard" funds. Out of those funds, 32,728 are listed as hedge funds, while 3,145 are listed as CTAs.

We follow previous studies and impose restrictions on the fund data set to deal with several sample biases. First of all, we exclude funds that either have not reported any data during the research period or have solely reported returns of 0% throughout their listing period in the database. Secondly, we account for survivorship bias and include both "live" and “graveyard funds” in our sample covering the period from January 1994 to December 2018 (see also Fung and Hsieh, 2000, Bali et al., 2012). We follow Fung and Hsieh (2000) and discard the first 12 months of each fund's return series data to eliminate back-fill bias. Effectively, this sets the constraint on a fund's minimum data requirement of at least 12 months of reported returns to be included in our data set. Finally, to mitigate the multi-period sampling bias, we follow Kosowski et al. (2007) and Bali et al. (2012) and require each fund to have at least 24 months of return observations. After imposing the above restrictions, our data set consists of 13,345 hedge funds over the period 1994 – 2018.[8]

Table 1 reports descriptive statistics for hedge funds for the period January 1994 – December 2018. In particular, the table provides the cross-sectional mean, median, standard deviation, 1st and 99th percentiles for the estimated coskewness (CS) and idiosyncratic skewness (IS), and fund characteristics including, returns, assets under management (AUM),

[8] We avoid incorporating any CTAs in our study as we are left only with a tiny number of them after considering the data restrictions.

incentive fee (InFee), management fee (ManFee) and minimum investment (MinInv) and redemption notice period.[9] To estimate the CS and IS measures for hedge funds, we use a rolling window regression for each month from January 1997 to December 2018. We start the regressions using each fund's first three years of monthly data (i.e., 36 months) from January 1994 to December 1996, as described in Section 2.

[Table 1 here]

The average hedge fund return is 0.44% per month. Both coskewness and idiosyncratic skewness means are negative for hedge funds reporting values of -0.04 and -0.01, respectively. Our descriptive statistics are similar to Bali et al. (2012) and the hedge fund industry in terms of assets under management, incentive fee, and management fee. In particular, hedge funds have a high dispersion in assets under management. While the mean hedge fund size is $310.64 million, the corresponding median size is only $37.5 million. This verifies the existence of very few hedge funds of enormous size. As for the incentive fees paid to fund managers for yielding superior performance, as a percentage of the funds' annual net profits above a specified threshold, the mean (median) incentive fee for hedge funds is 14.93% (20%). Those fees can climb up to 25% in some cases for a few hedged funds. Management fees' mean (median) is 1.44% (1.50%). The amount of minimum investment spent in dollars from hedge funds has a mean of $4.61 million, while its median is only $0.25 million. Finally, our examined hedge funds' mean redemption notice period is 37.90 days, while it can reach up to 120 days for a few funds.

[9] An incentive fee is a fixed fee (in percentage terms) of the fund's annual net profits above a specified hurdle rate. The management fee is a fixed fee (in percentage terms) on assets under management. The minimum investment is the minimum initial investment amount required by the fund for investment. The redemption period is the minimum number of days an investor should notify the fund before redeeming her/his investment amount from the fund.

4. Primary Empirical Findings

4.1. Fama – MacBeth regressions

4.1.1 Predictive power of skewness estimates controlling for stocks characteristics

To examine whether the cross-sectional variation in fund returns is explained by coskewness and idiosyncratic skewness, we run Fama-MacBeth (1973) cross-sectional regressions of hedge funds one-month-ahead excess returns on coskewness and idiosyncratic skewness, with and without controlling for individual fund characteristics.[10] We start with cross-sectional regressions of fund's returns on coskewness and idiosyncratic skewness estimates, without using any control variables over the period 1997 – 2018:

$$R_{i,t+1} = \lambda_0 + \lambda_t CS_{i,t} + \varepsilon_{i,t} \tag{7}$$

$$R_{i,t+1} = \lambda_0 + \lambda_t IS_{i,t} + \varepsilon_{i,t} \tag{8}$$

where, $R_{i,t+1}$ is the excess return of fund *i* in month *t+1* and $CS_{i,t}$ and $IS_{i,t}$ are the coskewness and idiosyncratic skewness estimates for month *t* as defined in (4) and (6), respectively. λ_0 and λ_t are the monthly intercepts and slope coefficients from the Fama-MacBeth (1973) regressions.

Table 2 (*Model 1* and *2*) reports the average slope coefficients of cross-sectional regressions from January 1997 to December 2018, while their corresponding Newey and West (1987) *t*-statistics are reported in parenthesis.

[Table 2 here]

The first column shows a positive and insignificant relationship between coskewness and hedge fund returns. The average slope coefficient is positive and close to zero with a Newey

[10] When we run the cross-sectional regressions, we winsorize the distributions of funds' returns, coskeweness, and volatility point estimates at the 0.5% and 99.5% levels to limit the effects of outliers.

and West t-statistic of 0.59. This is positive and statistically significant when it comes to the relationship between funds returns and idiosyncratic skewness. The average slope coefficient of hedge funds idiosyncratic skewness is 0.455, with an impressive t-statistic of 4.39. Such a high t-statistic is consistent with the recent conclusion of Harvey et al. (2016), who propose a new significance threshold of three, especially when controlling for data mining issues arising from multiple hypothesis testing. The above finding is also in line with Heuson et al. (2020), who also find a positive relationship between idiosyncratic skewness and hedge funds returns.

We now assess the explanatory power of coskweness and idiosyncratic skewness on funds' cross-section of future returns when controlling for several individual fund characteristics. We again run Fama-MacBeth (1973) cross-sectional regressions of funds' monthly one-month-ahead returns on coskewness and idiosyncratic skewness estimates, using a set of control variables found to be significant in explaining fund returns (see Bali et al., 2012; Bali et al., 2021). The set includes the past-month return, incentive fee, redemption period, minimum investment amount, and a dummy for lockup. The exact multivariate specification and its nested versions for cross-sectional regressions are given by

$$R_{i,t+1} = \lambda_0 + \lambda_{1,t}CS_{i,t} + \lambda_{2,t}IS_{i,t} + \lambda_{3,t}R_{i,t} + +\lambda_{4,t}InFee_{i,t} + \lambda_{5,t}RedPer_{i,t} + \lambda_{6,t}MinInv_{i,t} + \lambda_{7,t}DumLo_{i,t} + \varepsilon_{i,t} \quad (9)$$

where, $R_{i,t}$ is the current-month excess return of fund i representing the past-month return, $InFee_{i,t}$ is the incentive fee, $RedPer_{i,t}$ is the redemption period, $MinInv_{i,t}$ is the minimum investment amount and $DumLo_{i,t}$ is the dummy variable for lockup provisions of fund i in month t.[11]

[11] Lockup provisions exist when the fund does not allow withdrawal of initial investments from investors for a prespecified time.

Table 2 (*Model 3* to *Model 6*) displays the average slope coefficients of the coskewness, idiosyncratic skewness, lagged return, and the fund characteristics along with the Newey and West *t*-statistics (in parenthesis). *Model 3* assesses the explanatory power of both coskewness and idiosyncratic skewness on future funds' returns. The average slope coefficients reveal again positive and statistically significant predictive ability of idiosyncratic skewness on hedge fund returns at a 99% statistical significance level and positive but not significant predictive ability of coskewness on fund returns.

Models 4 to 6 findings show that controlling for funds lagged returns and individual characteristics does not change that idiosyncratic skewness is a robust determinant of cross-sectional differences in hedge funds returns. The average slope coefficient of coskewness is still statistically insignificant. The idiosyncratic skewness coefficient retains its predictive power and has magnitudes of 0.343 (Model 5) and 0.348 (Model 6) and Newey and West *t*-statistics of 4.65 and 4.75, respectively. When it comes to funds lagged return and individual predictability on the cross-section of funds returns, our findings are quite similar to those of the previous literature (see Brown et al., 1999, Liang, 1999, Agarwal and Naik, 2000; Edwards and Caglayan, 2001; Aragon, 2007). For instance, the average slopes on lagged returns, redemption period, minimum investment, and dummy lockup are positive and significant. Hence, hedge fund returns are evident in short-term persistence related to momentum effects and illiquidity premiums. The most considerable coefficients are hedge funds' lagged return and dummy lockup. The average slope on the former ranges from 0.109 to 0.110, and the slope on the latter ranges from 0.134 to 0.145. Their corresponding Newey and West *t*-statistics range from 6.80 to 7.05 and 3.48 to 3.78.

4.1.1 Predictive power of skewness estimates controlling for volatility, systematic and unsystematic risk

Bali et al. (2012) find that the predictive power of total volatility on hedge fund returns comes primarily from the systematic risk when they decompose total volatility in its systematic and unsystematic components. In particular, total volatility and systematic risk are highly significant in predicting the cross-section of future hedge fund returns. Hence, we repeat the Fama MacBeth (1973) cross-sectional regressions of funds one-month-ahead returns on coskewness and idiosyncratic skewness, but we control for total volatility systematic and unsystematic risk. Following Bali et al. (2012), we calculate the total volatility as the 36-month rolling window variance of funds returns and the systematic and unsystematic risk as to the 36-month rolling window variance of funds returns on an extended Carhart (1997) model, including the bond market and credit spread factors of Fung and Hsieh (2004) model. Hence, equation (9), including the exact multivariate specification and its nested versions, is transformed as:

$$R_{i,t+1} = \lambda_0 + \lambda_{1,t} CS_{i,t} + \lambda_{2,t} IS_{i,t} + \lambda_{3,t} Vol_{i,t-1} + \lambda_{4,t} SR_{i,t} + \lambda_{5,t} USR_{i,t} + \varepsilon_{i,t+1} \tag{10}$$

where, $Vol_{i,t}$ is the total volatility and $SR_{i,t}$ and $USR_{i,t}$ are the six-factor systematic and systematic risk at time t, respectively, as used in Bali et al, (2012). Table 3 reports the relevant average slope coefficients and their corresponding Newey and West (1987) *t*-statistics (in parenthesis) of the multivariate cross-sectional regressions for the period January 1997 - December 2018.

[Table 3 here]

The general picture reveals again the sound predictive power of idiosyncratic skewness estimates over funds returns even when controlling with other significant predictors such as the total volatility and systematic risk. *Models 1* to *3* provide similar information to that of

Table 3. Coskewness has positive but not significant explanatory power on hedge fund returns. Volatility has positive relationship with the cross-section of funds returns, but its average slope coefficient is not statistically significant when considering idiosyncratic skewness as the independent variable in our cross-sectional regression. For instance, the average slope coefficient estimate of volatility is 0.001, with Newey and West *t*-statistics ranging from 0.20 to 0.28. This critical finding shows that contrary to what was found by Bali et al. (2012), volatility is not a powerful determinant of the cross-sectional difference in funds returns when the hedge fund's idiosyncratic skewness is considered.

Models 4 to *6* report the cross-sectional regression results of hedge fund one-month-ahead returns on coskewness, idiosyncratic skewness, and systematic and unsystematic risk. Even though coskewness shows no predictive power on hedge funds' future returns, idiosyncratic skewness remains a strong predictor of fund returns even when controlling for systematic and unsystematic risk. The coefficients' estimates take values of 0.445 and 0.452 in *Models 5* and *6*, respectively. Their corresponding Newey and West *t*-statistics are 5.07 and 5.11, far above the 99% significance level threshold. Interestingly, the systematic risk, which is a robust and positive predictor of fund returns according to Bali et al. (2012), is not statistically significant when idiosyncratic skewness is also used as a predictive variable. The systematic risk coefficient has an average value of 0.004, and its corresponding *t*-statistics range from 0.48 to 0.74 in *Models 4* to *6*. On the other hand, and likewise in previous literature, there is a negative but insignificant link between unsystematic risk and hedge fund returns.

4.2. Portfolio analysis

4.2.1. Univariate portfolio analysis

We perform a univariate portfolio analysis to evaluate the relationship between coskewness and idiosyncratic skewness and fund returns. Starting from January 1997, we form equally – weighted quintile portfolios each month by sorting hedge funds based on their CS and IS estimates, where Quintile 1 contains funds with the lowest skewness estimates and Quintile 5 contains funds with the highest skewness estimates. Those portfolios are rebalanced every month to generate the time series of monthly returns from January 1997 to December 2018[12].

We evaluate both raw and abnormal time-series performance of the quintile portfolios. For the former, we compute each portfolio's average raw returns across the sample period, while for the latter, we estimate different factor models' alphas (i.e., abnormal returns) by running time-series regressions of each portfolio's excess returns on different factors. For that purpose, we use the Fung and Hsieh (2004) seven-factor model, a global Carhart (1997) four-factor model, and similar to Della Corte et al. (2020), a global Carhart (1997) model augmented with the betting–against–beta (Frazzini and Pedersen 2014) and the quality – minus – junk (Asness et al. 2014) factors.[13] To test the significance of the quintile portfolio's raw returns and alphas, we report *t*-statistics using Newey-West (1987) standard errors, which correct for heteroscedasticity and serial correlation.

[12] We have also run similar results of value-weighted portfolios. In that case, we weight each fund's monthly return, within the same quintile, with its corresponding CS and IS measures, respectively. The findings are similar to those of equally-weighted portfolios, and they are available upon request.

[13] We have obtained the monthly factor data for Fung and Hsieh's (2004) seven-factor model from David A, Hsieh's website (https://faculty.fuqua.duke.edu/~dah7/). The monthly global market, size, value, momentum, betting – against – beta, and quality – minus – junk factors have been obtained from AQR investment management research (https://www.aqr.com/Insights/Research). The market value factor is the value-weighted return on all available stocks minus the one–month Treasury bill rate. The size, value, and momentum factors have been constructed using value-weighted portfolios formed on size, book-to-market, and one-year return, respectively.

Table 4 reports the average monthly returns and the factor alphas across the coskewness and idiosyncratic skewness-sorted portfolios for hedge funds. The last row displays the difference between the two extreme quintile portfolios (i.e., Quintile 5 and Quintile 1) to assess the performance dispersion related to the skewness estimates.

[Table 4 here]

Looking at the coskewness portfolio results, the quintile portfolio with the highest coskewness (i.e., Quintile 5) generates an average statistically significant raw return of 0.447% (*t*-statistic = 2.80) per month, while the portfolio with the lowest coskewness (i.e., Quintile 1) delivers a significant average raw return of 0.538% (*t*-statistic = 2.69) per month. However, it does not exist a strong monotonic trend across the quintile portfolios, which is also justified by the insignificant negative return of the high-minus-low (5-1) quintile portfolio. The actual return of the 5-1 portfolio is -0.091%, and it has a Newey and West *t*-statistic of -1.00. The average alphas between the two extreme portfolios are always negative and insignificant. Hence, the insignificant correlation between coskewness and abnormal returns indicates that hedge fund managers possibly do not price coskewness risk at a general level.

We now concentrate on the idiosyncratic skewness–sorted portfolios. Average raw returns and the Fung and Hsieh (2004), Charhart (1997), and six-factor alphas are positive and statistically significant for most quintiles. Only for Quintile 1, the positive portfolio alphas are marginally or not statistically significant across all factor models. For instance, Quintile 1 delivers a Fung Hsieh (2004) alpha of 0.151% (*t*-statistic = 1.59) per month and Quintile 5 yields 0.235% per month (*t*-statistic = 3.16). More importantly, we observe a monotonic upward trend of raw returns and factor alphas as we move from low to high idiosyncratic skewness portfolios. When it comes to extreme portfolio spread (i.e., 5-1 portfolio), raw return and factor alphas are all positive and statistically significant, with the raw return being 0.271%

per month and having a Newey and West *t*-statistic of 4.06. The factor model alpha values range from 0.234% to 0.303% per month, and their corresponding *t*-statistics span from 2.89 to 3.71, respectively. Hence the positive relationship between idiosyncratic skewness and fund future returns probably suggests that fund managers hold undiversified portfolios because of managerial skill or lottery-type behaviors. In the following section, we investigate if managerial skill is the main determinant of such an empirical finding.

4.2.2 Bivariate portfolio analysis

We proceed with a bivariate quintile portfolio analysis for coskewness and idiosyncratic skewness. We perform a quintile portfolio analysis for coskewness by controlling for idiosyncratic skewness and vice versa. Such an experiment is similar to double-sorting the hedge funds based on both skewness measures. We start with coskewness bivariate portfolio analysis and for every month from January 1997 to December 2018. We first sort funds' returns into quintiles based on their rolling estimate of idiosyncratic skewness (IS). Then, within each IS ranked quintile portfolio, we sort funds further into subquintiles based on their coskewness rolling estimate (CS).[14] Similarly, by changing the sorting order between coskewness and idiosyncratic skewness, we form quintile portfolios for every month by first sorting funds' returns into quintiles based on their CS estimates and then by sorting further into subquintiles based on their IS estimates, within each CS ranked portfolio. This procedure generates subquintile portfolios of hedge funds spreading around CS (IS) with almost identical IS (CS) values.

[14] This equivalent of constructing 25 subquintile portfolios, where $Q_{i,j}$ is the *j*th sorted *CS* portfolio within *i*th sorted *IS* portfolio ($i = 1,2, \dots ,5; j = 1,2, \dots ,5$) (see also Bali et al, 2012).

Table 5 reports the monthly average returns and the Fung and Hsieh (2004), global Carhart (1997), and six-factor alphas of the double-sorted portfolios and the long-short portfolio of the highest and lowest quintile along with their corresponding Newey and West adjusted *t*-statistics. Panel A of the same table reports the relevant average returns and alphas of coskewness quintile portfolios while controlling for idiosyncratic skewness. For example, $CS, 1$ represents the lowest CS sorted funds' quintile portfolio within the IS sorted quintiles. The same panel also presents the average CS values within each IS sorted quintile Panel B presents the relevant findings for idiosyncratic skewness portfolios while controlling for coskewness.

[Table 5 here]

Panel A reveals a monotonic downward trend of average CS portfolio returns as we move from CS,1 to CS,4 quintiles, but this terminates in $CS, 5$ portfolio, which generates a higher return than CS,4. The average monthly return of the difference between $high$ and low CS quintile portfolios (i.e., *CS,5 – CS,1*) is negative but not statistically significant. Its average return is -0.067%, with a *t*-statistic of -0.74. The Fung and Hsieh (2004) and Carhart (1997) factor model alphas of the same portfolio are also negative and statistically insignificant. For instance, the relevant alpha Fung and Hsieh (2004) estimate is -0.040% with a Newey and West *t*-statistic of -0.43. When it comes to the alpha of the six-factor model, this is positive but again not statistically significant. The average six-factor alpha is 0.024%, with a *t*-statistic of 0.21. The above findings are similar to those obtained from our Fama and MacBeth regressions and univariate portfolio analysis. Even though there is a trade-off between coskewness and future hedge fund returns, which is consistent with the generic literature around coskewness, such a trade-off is not statistically significant in the case of the hedge funds when we simultaneously control for idiosyncratic skewness.

The relevant idiosyncratic skewness quintile portfolio analysis presented on Panel B of the same table reports that as we move from *IS,1* to *IS,5* quintiles, the average return on the *IS* increases monotonically. The average return increases from 0.306% to 0.576%. The average return difference between *high* and *low IS* funds is positive and statistically significant. The corresponding long-short *IS* portfolio of hedge funds delivers an average monthly return of 0.269% with a Newey and West *t*-statistic of 4.06. Such evidence supports that the positive relationship between idiosyncratic skewness and future hedge fund returns remains significant after controlling for coskewness, which is similar to that found in our Fama and MacBeth regressions. The generated factor alphas of the *IS,5-IS,1* portfolio are positive and statistically significant. For example, the Fung and Hsieh (2004) factor-alpha difference between *IS,5,* and *IS,1* quintile of hedge fund returns is 0.243% with a Newey and West *t*-statistic of 4.06. Likewise, the global Carhart (1997) and six-factor alphas of the *IS,5 – IS,1* portfolio is 0.247% and 0.324%, with respective *t*-statistics of 3.13 and 4.06, respectively. The generated alpha values' significance indicates that the above models' factors do not explain the positive link between the idiosyncratic skewness and the cross-section of hedge fund returns. Thus, the yielded alphas should reflect managerial skill, which we investigate further in our next section.

5. What explains the Skewness-Fund Performance Relation?

This section tries to economically interpret our main finings related to skewness premia and investigate some of their potential determinants. We examine two possible sources of coskewness, and idiosyncratic skewness in funds returns other than those of the assets held by the funds. First, we use a skill-based explanation assigning the idiosyncratic skewness premia and potential coskewness premia of hedge funds to managerial skill. Second, we assess whether successful market timing explains a positive relationship between hedge fund returns and coskewness and idiosyncratic skewness.

5.1. Skill-Based Explanation

Following Chen et al. (2020), who use Titman's and Tiu's (2011) hedge fund skill measure to test whether the significant returns generated by high sentiment beta hedge funds reveal managerial skill, we perform a similar experiment to assess whether coskewness and idiosyncratic skewness risk premia are related to managerial skill. Titman and Tiu (2011) support that high-skilled managers, being more confident in their abilities to generate alphas from active strategies, will have lower exposure to systematic factors so that their funds reveal a lower R-squared in the relevant regressions. On the other hand, low-skilled managers will have greater exposure to systematic factors so that they will have a higher R-squared. For that purpose, we employ the Fung and Hsieh (2004) seven-factor model, and for each fund, we run a rolling window regression of 36 monthly observations on the model's factors to obtain its corresponding R-squared for each month. We categorize a fund as high (low) skilled if its R-squared is below (above) the cross-sectional median for each month. Then we form quintile portfolios for every month by sorting low and high-skilled hedge funds based on their coskewness and idiosyncratic skewness estimates, similar to our portfolio analysis in Section 4.2.1.

Panel A of Table 6 presents the average one-month-ahead returns and the factor alphas across the coskewness and idiosyncratic skewness-sorted portfolios of high-skilled hedge funds. The last row again reports the difference between the two extreme quintile portfolios (i.e., Quintile 5 and Quintile 1) and the corresponding Newey and West *t*-statistics. Panel B of the same table demonstrates the relevant findings for low-skilled portfolios.

[Table 6 Here]

At first glance, it is evident that our findings are far more robust and statistically significant for the high-skilled compared to those of low-skilled hedge funds, especially by looking at the performance of the high-minus-low portfolios. Interestingly, the returns of the coskewness-sorted portfolios of high-skilled funds (i.e., Panel A) move almost monotonically downward as we move from Quintile 1 to Quintile 5. The *5-1* portfolio generates a statistically significant return of -0.189% with a Newey and West *t*-statistic of -1.97, while the generated alphas range from -0.125% to -0.170%, and they are marginally statistically significant, except the case of six-factor model alpha. On the contrary, the monotonic downward trend of coskewness-sorted portfolios' returns does not exist for the case of low-skilled hedge funds. More importantly, the generated returns of the spread portfolio are insignificant though negative.

Interestingly, the above results support that skilled hedge fund managers significantly price negative coskewness risk both in economic and statistical terms over and above several factors used by the hedge fund literature. A finding consistent with the coskewness pricing model of Kraus and Litzenberger (1976) suggesting that managers should not be able to produce both desirable alpha and coskewness. Hence, skilled fund managers require a high premium for having a negative coskewness with the market, while this is not the case for low-skilled fund managers as in Panel B of Table 6.

The relevant results of the portfolios of funds sorted based on their idiosyncratic skewness reveal a more pronounced picture for high-skilled against low-skilled funds when comparing their quintile portfolios' performance. The quintile portfolios of high-skilled funds report a robust upward trend as we move from Quintile 1 to Quintile 5 in terms of both returns and factor alphas. This trend of portfolio returns results in the high-minus-low portfolio yielding a positive return of 0.277% with an outstanding Newey and West *t*-statistic of 4.70. The factor alphas span from 0.223% to 0.267%, while their corresponding *t*-statistics range from 3.34 to 3.88. Such a positive relationship between idiosyncratic skewness and future hedge fund returns weakens when it comes to the relevant quintile portfolio performance of low-skilled funds. The monotonic upward trend among portfolios still exists, but the difference between high and low portfolios is marginally significant here. For instance, the one-month-ahead return of the spread portfolio is 0.182% with a *t*-statistic of 1.77, while the factor model alphas are insignificant except from the case of the six-factor model alpha, which has a value of 0.228% and a *t*-statistic of 2.08. Hence, we prove that the predictive power of idiosyncratic skewness over future fund returns is probably a result of managerial skill than the gambling preferences of fund managers. High-skilled managers successfully exploit the upside potential of positively skewed assets to generate alpha compared to low-skilled managers.

5.2. Time-Varying Market Exposures

Second, we try to discover to what extent hedge fund returns' coskewness and idiosyncratic skewness are determined by market timing, which is also a source of *skill* (see Kacperczyk et al., 2014). To achieve this, we follow Back et al. (2018) and decompose coskewness and idiosyncratic skewness into a component due to time-varying market exposures and a component due to other factors. First, we define the amount of skewness estimates that is due to time-varying exposures using Ferson and Schadt's (1996) regression model of a fund on the market factor and the product of the market factor with lagged public information:

$$r_{i,t} - r_{f,t} = \alpha_i + b_{i,1}\left(r_{M,t} - r_{f,t}\right) + b'_{i,2}(z_{t-1})\left(r_{M,t} - r_{f,t}\right) + u_{i,t} \qquad (11)$$

where z_{t-1} is a vector of deviations of the public conditioning variables from their unconditional means. This vector consists of the dividend yield of the CRSP value-weighted NYSE and AMEX stock index over the previous 12 months, the 1-month Treasury-bill yield, the term spread (i.e., constant maturity 10-year Treasury bill less 3-month Treasury bill yield), and the corporate bond default spread (Baa-rated minus Aaa-rated corporate yields).[15] We then compute the coskewness of the excess return, $\alpha_i + u_{i,t}$, relative to market return and the idiosyncratic skewness of the same excess return, as described in Section 2. Those are the coskewness and idiosyncratic skewness estimates of a hedge fund not due to time-varying betas based on public information.[16] To calculate the part of the coskewness (idiosyncratic skewness) due to the time-varying betas, we simply subtract from the total coskewness (idiosyncratic skewness), used in the previous sections, the *non*-time-varying estimate.

We run cross-sectional regressions to assess whether market timing is a possible source of coskewness and idiosyncratic skewness in funds returns. We follow Back et al. (2018) and run cross-sectional regressions of the exact skewness estimates on funds future returns and, on a constant, as presented in Table 7. A fund is a good market timer based on public information if it creates a positive return and a positive skewness estimate and vice versa. Hence, we expect both coskewness and idiosyncratic skewness to be positively related to market timing because funds that successfully time the market generate return profiles, which are convex in the market return (see Treynor and Mazuy, 1996).

[Table 7 Here]

Table 7 reveals that, indeed, there is a positive relationship between funds' future returns and the coskewness and idiosyncratic skewness estimates based on time-varying betas on public information. For instance, time-varying coskewness is positively related to future returns, and the corresponding coefficient is statistically significant at the 5% statistical level.

[15] We collect dividend yields from CRSP, while the other conditioning variables are from the Federal Reserve Economic Data (FRED). Following Back et al. (2018) and Barras et al. (2010), we exclude the January dummy variable employed by Ferson and Schadt (1996) in the vector of conditioning variables, and our findings remain the same without the exclusion.
[16] This component may include market timing activities based on private information and other actions.

On the other hand, there is a significantly negative correlation between the future fund returns and *non*-time-varying coskewness as this is implied by the Newey and West *t*-statistic of -3.15 of the relevant coefficient. Such a strongly negative relationship indicates that any market timing ability based on private managerial signals, resulting in positive correlation, is dominated by alpha-generating activities that create negative coskewness, in other words, activities that require a reward for being exposed to negative skewness with the market. Additionally, the undesirable coskewness per unit of future return is even higher for the hedge fund returns not due to public information market timing, than for the total returns. Such a finding is also justified because the correlation of the total coskeweness with fund returns is negative but marginally statistically significant (i.e., *t*-statistic of -1.81), consistent with our previous sections' results.

The cross-sectional regression results of the time-varying and *non*-time-varying components of idiosyncratic skewness on future hedge fund returns reveal a positive and significant relationship among both components and the returns. As anticipated, the time-varying idiosyncratic skewness is positively correlated with the one-month-ahead fund returns as well as it is significant at the 99% statistical level (i.e., *t*-statistic of 3.36). Interestingly, there is an even stronger positive relationship between the *non*-time-varying idiosyncratic skewness and fund returns. The magnitude of the relevant coefficient and its Newey and West *t*-statistic is larger than that of the corresponding coefficient of the time-varying component (i.e., *t*-statistic of 4.65). Such a finding shows that any market timing ability is dominated by alpha-generating activities related to holding undiversified portfolios and taking advantage of positively skewed assets. In other words, funds probably include market timing activities based on private information and other actions that generate positive alpha regarding idiosyncratic skewness, which possibly needs further investigation. Finally, the corresponding coefficients of the constant term are most of the time negative and statistically significant.

6. Matrix completion and skewness premia

This section uses a *matrix completion* approach to deal with missing hedge fund data. This is common in hedge funds datasets as many funds operate for short periods before liquidating, while new funds frequently appear, leading to unbalanced panels. We apply a *matrix completion* method from the machine learning literature to recover missing entries in the matrix of funds returns and perform the cross-sectional regression and portfolio analysis experiments again.

6.1 Matrix completion method

We adopt the *soft-impute matrix completion* method of Mazumder et al. (2010) suitable for recovering data of a sizeable $N \times T$ matrix X (e.g., $N, T \approx 10^6$-10^8) for which only a relatively small number of entries are observed. In *matrix completion* methods, the problem is expressed as learning an unknown parameter (i.e., an $N \times T$ matrix Z) with high dimensionality, based on a few observations. In other words, Z represents an $N \times T$ low-rank matrix. Additionally, we can view the observed data of matrix X as the corresponding data from matrix Z contaminated with noise.

We can also define an $N \times T$ matrix $P_{\Omega}(X)(i,j)$ whose (i,t) th element is $X_{i,j}$ if $(i,j) \in \Omega$, otherwise is zero, in which case Ω denotes the indices of the observed series. In other words, $P_{\Omega}(X)$ is the projection of X onto the observed series. Similarly, we can also define as $P_{\Omega}{}^{\perp}(X)$ the complementary projection, where $P_{\Omega}(X) + P_{\Omega}{}^{\perp}(X) = X$. Then we should consider the following optimization problem:

$$\widehat{Z_{\lambda}} = \underset{Z}{\text{minimize}}\ \frac{1}{2}\|P_{\Omega}(X) - P_{\Omega}(Z)\|^2 + \lambda\|Z\|_* \tag{11}$$

where, $\|Z\|_*$ denotes the nuclear norm of Z and $\lambda \geq 0$ is a regularization parameter controlling for the nuclear norm of the minimizer $\widehat{Z_\lambda}$. The solution is given by calculating the truncated singular value decomposition (SVD) for X (i.e., a low-rank SVD of a matrix).[17] Specifically, the exact procedure efficiently provides a sequence of solutions for equation (11) for different values of λ based on *warm-starts* (see also Mazumder et al., 2010). The algorithm repeatedly replaces the missing entries with the current estimate and then updates the estimate through updating the SVD using the *complete* data matrix. In this way, the method can easily handle matrices of large dimensions by exploiting the problem structure. This means that an SVD computation is performed at every iteration, in which the specification decreases the value of the objective function towards its minimum, and at the same time, gets closer to the set of optimal solutions. This computation is performed for the number of simulations until convergence to the optimal solution of the objective function or a certain tolerance threshold is achieved. The sequence of solutions converges asymptotically to optimal as the number of optimization iteration approaches infinity.[18] We use 10,000 simulations and a convergence threshold of 0.0001for our application.[19]

Because the low-rank matrix $\widehat{Z_\lambda}$ in (11) is achieved by penalizing the singular values of Z, the number of singular values retained may exceed the actual rank of the matrix. In such cases, we need to unshrink the chosen outputs, which might permit a lower-rank solution (see Mazumder et al., 2010). To accomplish such a process, we can insert a matrix $M_{r_\lambda \times r_\lambda}$, where r_λ is the rank of Z_λ as estimated by (11), to secure a lower training error for the same rank. Likewise

[17] This is given by $\hat{Z} = UD_\lambda V'$ where U is an $N \times r'$ matrix, V is an $T \times r'$, and $D_\lambda = diag[(d_1 - \lambda)_+, \dots, (d_r - \lambda)_+]$, with $r' = \min\,(N, T)$ (see also, Mazumder et al., 2010).

[18] For more information on the convergence analysis and the *minimum* convergence rate of the method, see Mazumder et al., (2010).

[19] We have also tried different combinations of iterations and convergence (e.g., 1,000 and 0.00, respectively). The empirical findings remain at least qualitatively the same.

calculating the SVD in (11), we consider again two matrices U and V, each of rank r_λ, and we can then solve the Frobenius norm of an affine transformation of M as

$$\widehat{M} = \arg\min_{M} \ \|P_\Omega(X) - P_\Omega(UMV')\|^2 \qquad (12)$$

where $\widehat{Z_\lambda} = U\widehat{M}V'$

The solution of the above objective function can decrease the training error obtained by solving (11) for the same rank.

6.2 Fama – MacBeth regressions

Again, we run the same Fama-MacBeth (1973) cross-sectional regressions of hedge funds one-month-ahead excess returns on coskewness and idiosyncratic skewness, with and without controlling for individual fund characteristics as those in section 4.1. This time though, the hedge fund returns missing data has been recovered using the *matrix completion* method described above, and so the estimated systematic and idiosyncratic skewness and volatility measures have been calculated accordingly.[20] Tables 8 and 9 report the relevant results.

[Table 8 Here]

[Table 9 Here]

Models 1 and *2* of Table 8 present the univariate cross-sectional regressions' coefficients of one-month ahead returns on coskewness and idiosyncratic skewness, respectively. Interestingly, the coskewness coefficient is now negative and statistically significant at a 5% significance level. The coefficient's value and corresponding Newey and West *t*-statistic are -0.009 and -2.70, respectively. The estimated idiosyncratic skewness coefficient reveals that the

[20] The matrix completed hedge fund returns data has also been winsorized for our cross-sectional regressions.

positive relationship between hedge funds' future returns and idiosyncratic skewness remains highly significant at the 1% level. However, the magnitude of the coefficient has fallen to 0.175 compared to the relative findings of Table 2. *Models 3* to *5* report the multivariate cross-sectional regressions' findings with and without controlling for past return and funds characteristics. Both systematic and idiosyncratic skewness coefficients remain significant at 5% and 10%, respectively. For instance, the former range from -0.007 to -0.009 with *t*-statistics varying from -2.44 to -2.63, while the latter range from 0.130 to 0.184 with *t*-statistics varying from 4.02 to 4.57. Lagged returns' and redemption period's coefficients and their corresponding Newey and West *t*-statistics are similar to Table 2.

On the other hand, management fees' coefficients are now marginally negative, while those of lockup dummy show more than fifty percent drop in value compared to Table 2. Both coefficients retain their statistical significance at a 1% level. Based on those findings, we can conclude that both systematic and idiosyncratic skewness have predictive power on hedge fund returns when a complete hedge fund dataset is considered and even when controlling for funds' significant characteristics. The negative and significant coefficient of coskewness shows a trade-off between funds returns and coskewness. In other words, there is a coskewness cost for future returns for hedge funds. The positive and significant coefficient of idiosyncratic skewness probably reveals managerial skill, as the previous sections explained.

The above systematic and idiosyncratic skewness premia are also robust when controlling for total volatility, systematic and unsystematic risk, as shown in Table 9. The corresponding cross-sectional regressions (i.e., *Models 1* to *6*) reveal a negative and significant coefficient for systematic skewness at 5% and a positive and significant coefficient for idiosyncratic skewness at 1% level. However, the magnitude of the former is slightly reduced, while the latter remains the same on average, compared with the relevant values of Table 8. It is noteworthy that total volatility has predictive power over funds returns, with the regression coefficients varying from

0.011 to 0.013 and the corresponding *t*-statistics ranging from 4.25 to 4.55 (i.e., *Models 1* to *3*). When we break down the total volatility in its systematic and idiosyncratic components, only the systematic risk coefficient is positive, which is significant at 1% level. The coefficient takes values from 0.025 to 0.028 (i.e., *Models 4* to *6*). Those results are in line with the findings of Bali et al. (2012), who found that both total and systematic risk show predictive power over hedge funds returns.

6.3 Portfolio analysis

We replicate the univariate portfolio analysis presented in Section 4.2 for the fully recovered hedge fund returns matrix to evaluate the relationship between coskewness and idiosyncratic skewness and fund returns. Table 10 displays the relevant evidence.

[Table 10 here]

The findings are consistent with those obtained from Fama and MacBeth (1973) cross-sectional regressions reported in the previous section. In terms of coskewness-sorted portfolios of hedge funds, there is almost a monotonic downward trend for both raw returns and factor alphas as we move from Quintile 1 to Quintile 5. The lowest coskewness portfolio (i.e., Quintile 5) yields a significantly higher average excess return relative to the portfolio of funds with the highest coskewness. The spread 5-1 for the equally-weighted returns equals a statistically significant -0.183% per month with a Newey and West *t*-statistic of -2.48. The picture is similar when we estimate the abnormal time-series performance of the high-minus-low (5-1) portfolio. All generated alphas of the spread portfolio are negative and statistically significant. The 5-1 portfolio yields an abnormal return of -0.142% (*t*-statistic = -2.16) under the Fung and Hsieh (2004) model, -0.153% (*t*-statistic = -2.29) under the global Carhart (1997) model and -0.138 (*t*-statistic = -1.79) under the six-factor model. These results support that fund managers require a premium for holding assets with negatively coskewed returns, while they can accept a lower return for positively skewed assets. Hence the negative coskewness risk is significantly priced on hedge funds in economic and statistical terms over and above several risk factors.

Focusing on the idiosyncratic skewness–sorted portfolios under *matrix completion*, our findings are similar to Table 4. There is a monotonic upward trend of raw returns and alphas as we move from low to high idiosyncratic skewness portfolios. Quintile 5 portfolio generates a significantly higher average excess return than Quintile 1 portfolio. Regarding portfolio spread (i.e., 5-1 portfolio), raw return and factor alphas are all positive and statistically significant but lower in values than the relevant ones in Table 4. The spread portfolio yields a raw return of 0.180% (*t*-statistic = 4.25), an abnormal return of 0.165% (*t*-statistic = 3.80) under the Fung and Hsieh (2004) model, 0.181% (*t*-statistic = 3.43) under the global Carhart (1997) model and 0.226% (*t*-statistic = 4.15) under the six-factor model. Thus, we validate the potential managerial skill of hedge fund managers to hold undiversified portfolios when it comes to idiosyncratic skewness.

7. Conclusion

We examine the link of the cross-sectional variation in hedge fund returns to the differences in funds' systematic and idiosyncratic skewness. We use up-to-date and fund-suited methodologies for measuring the coskewness and idiosyncratic skewness of hedge funds. Those methods consider that hedge funds operate in different horizons and control for outliers in hedge fund returns, which can strongly bias skewness estimates. We find that idiosyncratic skewness predicts hedge fund returns in the cross-section. Hedge funds with high idiosyncratic skewness significantly outperform those with low idiosyncratic skewness in average returns and factor alphas. The return spread between the top and bottom quintiles of hedge funds ranked with idiosyncratic skewness is as large as 0.27% per month, while the factor alphas range from 0.23% to 0.30% per month on a risk-adjusted basis. Our Fama-MacBeth cross-sectional regressions verify the above finding even when controlling for hedge fund characteristics and total variance and systematic risk. We show that idiosyncratic skewness is a more powerful predictor of funds' returns than the total variance and systematic risk.

The explanatory power of idiosyncratic skewness on hedge funds returns is verified even when we adopt a matrix completion technique from the machine learning literature to recover missing entries of fund returns. Hedge fund databases usually suffer from the short horizon or missing time-series elements. Hence, we try to recover a low-rank matrix of funds returns based on the observed series and repeat our asset pricing experiments. Interestingly, systematic skewness premium in hedge fund returns is also revealed in that case in both portfolio analysis and Fama and MacBeth regressions. The return spread between the top and bottom quintiles of hedge funds ranked with coskewness is -0.18% per month, while the factor alphas range from -0.138% to -0.153% per month on a risk-adjusted basis.

We examine two economic rationales of our findings. The first one is a skill-based explanation shedding light on whether managerial skill drives the skewness risk premium. Not only the positive relationship between idiosyncratic skewness and fund returns is much more robust, but we also verify a coskewness risk premium among high-skilled hedge funds. The high-minus low coskewness portfolio return is significantly negative for high-skilled hedge funds, generating an average of -0.19% per month. Hence, skilled hedge funds demand compensation for negative coskewness, while their managerial skill is probably the reason for generating alpha by taking advantage of the upside potential of assets with positive idiosyncratic skewness. The second determinant of our findings is market timing. Funds that are successful market timers create convex return profiles with the market and should have positive coskewness and idiosyncratic skewness with the market. We decompose both coskewness and idiosyncratic skewness into two parts, one that is due to time-varying market exposures and one that is due to other factors. We find significantly positive coefficients for both coskewness and idiosyncratic skewness due to time-varying exposure in our cross-sectional regressions against expected hedge fund returns. The relevant coefficients related to other factors (e.g., alpha-generating activities) reveal even stronger relationships between the funds' returns and skewness estimates, but the coskewness coefficient is negative this time. Such a find suggests the dominance of funds alpha-generating activities over successful market timing.

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Table 1. Descriptive Statistics for Hedge Funds and CTAs (sample period 1994 – 2018)

The table reports the descriptive statistics of monthly returns and fund characteristics between January 2014 and December 2018 for the hedge funds sample examined. In particular, the number of funds used (N), including live and graveyard funds, the cross-sectional mean, median, standard deviation (Std. Dev.), 1st and 99th percentiles, are displayed. The fund characteristics include coskewness (CS), idiosyncratic skewness (IS), size (average monthly AUM), incentive fee (InFee), management fee (ManFee), and minimum investment (MinInv). Coskewness (CS) and idiosyncratic skewness (IS) have been estimated via a rolling window of 36 monthly observations.

	N	Mean	Median	Std. Dev.	1st percentile	99th percentile
Funds (Number of funds: 10,516)						
Average monthly return (%)	10,516	0.44	0.49	4.78	-12.04	12.28
CS	10,516	-0.04	-0.01	0.33	-1.18	0.87
IS	10,516	-0.01	-0.01	0.20	-0.49	0.48
Average monthly AUM (millions $)	10,516	310.64	37.50	2,652.20	0.02	3,9545.00
InFee (%)	10,516	14.93	20.00	7.89	0.00	25.00
ManFee (%)	10,516	1.44	1.50	0.57	0.00	3.00
MinInv (millions $)	10,516	4.61	0.25	113.80	0.00	25.00
Redemption Notice Period (days)	10,516	37.90	30.00	32.35	0.00	120.00

Table 2. Fama – MacBeth regressions of one-month ahead hedge fund returns on skewness estimates and control variables

This table reports the coefficient estimates from Fama – MacBeth (1973) cross-sectional regressions of one – month ahead funds excess returns on a constant, the funds' estimated coskewness (CS) and idiosyncratic skewness (IS), and a set of control variables (i.e., past month's return, incentive fee, redemption period, minimum investment and dummy for lockup). Point estimates for funds are winsorized at 0.5% and 99.5%. We have run the Fama – MacBeth (1973) regressions for each month and the full sample period January 1994 – December 2018. Coskewness (CS) and idiosyncratic skewness (IS) have been estimated via a rolling window of 36 monthly observations. Newey-West corrected *t*-statistics are presented in parentheses. Coefficients that are significant at 1%, 5% and 10% are denoted by ***, **, *, respectively.

Variable	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
CS	0.002		0.003	0.001		0.002
	(0.59)		(0.82)	(0.27)		(0.52)
IS		0.455 ***	0.464***		0.343***	0.348***
		(4.39)	(4.51)		(4.65)	(4.75)
Lret				0.110***	0.109***	0.109***
				(6.85)	(7.05)	(6.80)
InFee				0.000	-0.000	-0.000
				(0.43)	(-0.28)	(-0.14)
RedPer				0.001 **	0.001***	0.001***
				(2.53)	(2.74)	(2.68)
MinInv				0.000**	0.000**	0.000***
				(2.57)	(2.50)	(2.62)
DumLo				0.145***	0.137***	0.134***
				(3.78)	(3.55)	(3.48)
Adj. R^2	0.01	0.005	0.02	0.09	0.08	0.10

Table 3. Fama – MacBeth regressions of one-month ahead hedge fund returns on skewness estimates and volatility

This table reports the coefficient estimates from Fama – MacBeth (1973) cross-sectional regressions of one – month ahead funds excess returns on a constant, the funds' estimated coskewness (CS) and idiosyncratic skewness (IS), and a set of risk variables (i.e., total volatility, systematic and unsystematic risk). Point estimates for funds are winsorized at 0.5% and 99.5%. We have run the Fama – MacBeth (1973) regressions for each month and the full sample period January 1994 – December 2018. Coskewness (CS), idiosyncratic skewness (IS), total volatility (Vol), systematic (SR), and unsystematic risk (USR) have been estimated via a rolling window of 36 monthly observations, according to Bali et al. (2012). Newey-West corrected *t*-statistics are presented in parentheses. Coefficients that are significant at 1%, 5% and 10% are denoted by ***, **, *, respectively.

Variable	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
CS	0.002		0.003	0.001		0.001
	(0.61)		(0.79)	(0.18)		(0.38)
IS		0.438***	0.446***		0.445***	0.452 ***
		(4.87)	(4.93)		(5.07)	(5.11)
Vol	0.001	0.001	0.001			
	(0.36)	(0.28)	(0.20)			
SR				0.004	0.004	0.004
				(0.74)	(0.61)	(0.48)
USR				-0.002	-0.002	-0.003
				(-1.05)	(-0.88)	(-1.23)
Adj. R^2	0.07	0.06	0.07	0.09	0.08	0.10

Table 4. Univariate portfolios sorts of hedge funds by skewness estimates

This table presents the one-month ahead average excess abnormal returns of quintile portfolios of funds sorted by coskewness (CS) and idiosyncratic skewness (IS) estimates. Quintile portfolios are formed every month from January 1997 to December 2018 by sorting funds based on their *CS* and *IS* values estimated via a rolling window of 36 monthly observations. Quintile 1 is the portfolio of funds with the lowest *CS/IS*, and Quintile 5 is the portfolio with the highest *CS/IS*. We report the equally–weighted returns for each quintile portfolio and a high – minus – low quintile portfolio (*5 – 1*) and their generated alphas with respect to Fung and Hsieh, Carhart, and six–factor models, respectively. Newey-West adjusted *t*-statistics for the *5 – 1* portfolio returns are reported in parentheses. *,**,*** denote the statistical significance at 10%, 5% and 1%, respectively.

	CS				IS			
Quintiles	Raw Returns	FH alpha	Carhart alpha	Six-factor alpha	Raw Returns	FH alpha	Carhart alpha	Six-factor alpha
1	0.538***	0.276***	0.271***	0.319***	0.324**	0.151	0.127	0.155*
	(2.69)	(2.65)	(2.80)	(3.04)	(2.30)	(1.59)	(1.43)	(1.65)
2	0.463***	0.281***	0.259***	0.274***	0.398***	0.203**	0.197***	0.261***
	(3.50)	(3.65)	(3.72)	(4.00)	(2.81)	(2.50)	(2.73)	(3.37)
3	0.422***	0.275***	0.252***	0.281***	0.446**	0.246***	0.235***	0.281***
	(3.64)	(3.86)	(3.72)	(4.23)	(3.09)	(3.08)	(3.30)	(3.63)
4	0.419***	0.259***	0.241***	0.284***	0.527***	0.314***	0.303***	0.329***
	(3.57)	(3.47)	(3.57)	(3.83)	(3.66)	(3.82)	(4.31)	(4.75)
5	0.447***	0.209***	0.199***	0.324***	0.595***	0.386***	0.361***	0.458***
	(2.80)	(2.00)	(2.28)	(3.04)	(4.08)	(4.91)	(5.01)	(6.17)
5-1	-0.091	-0.066	-0.072	0.004	0.271***	0.235***	0.234***	0.303**
	(-1.00)	(-0.71)	(-0.73)	(0.03)	(4.06)	(3.16)	(2.89)	(3.71)

Table 5. Bivariate portfolios sorts of hedge funds by skewness estimates

This table presents the monthly average excess abnormal returns of quintile funds portfolios double – sorted by coskewness (CS) and idiosyncratic skewness (IS) estimates. In Panel A, equally-weighted quintile portfolios are formed every month from January 1997 to December 2018 by first sorting funds based on their *IS* values estimated via a rolling window of 36 monthly observations. Then, within each *IS* portfolios, funds are sorted into subquintiles based on their *CS* values estimated via a rolling window of 36 monthly observations. Quintile CS,1 is the portfolio of funds with the lowest *CS* within each *IS* quantile portfolio and Quintile CS,5 is the portfolio of funds with the highest *CS* within each *IS* quantile portfolio. Panel B repeats the same procedure for quintile portfolios of funds sorted by *IS* after controlling for *CS*. The last rows present the equally weighted returns of a high – minus – low (*5 – 1*) quintile portfolio and their resulting alphas with respect to Fung and Hsieh, Carhart, and six–factor models, respectively. Newey-West adjusted *t*-statistics for the *5 – 1* portfolio returns are reported in parentheses. *,**,*** denote the statistical significance at 10%, 5% and 1%, respectively.

Panel A: *Funds sorted by CS after controlling for IS*			Panel B: *Funds sorted by IS after controlling for CS*		
CS Quintiles	Average *CS* in each *IS* quintile	Next month average Returns	IS Quintiles	Average *IS* in each *CS* quintile	Next month average Returns
CS,1	-0.050	0.507	IS,1	-0.280	0.306
CS,2	-0.015	0.462	IS,2	-0.106	0.403
CS,3	-0.005	0.425	IS,3	-0.001	0.449
CS,4	0.004	0.423	IS,4	0.103	0.523
CS,5	0.036	0.440	IS,5	0.281	0.576
CS,5 – CS,1		-0.067	IS,5 – IS,1		0.269***
		(-0.74)			(4.06)
FH alpha		-0.040	FH alpha		0.243***
		(-0.43)			(3.22)
Carhart alpha		-0.045	Carhart alpha		0.247***
		(-0.47)			(3.13)
Six-factor alpha		0.024	Six-factor alpha		0.324***
		(0.21)			(4.06)

Table 6. Univariate portfolios sorts of high and low skilled hedge funds by skewness estimates

This table presents the one-month ahead average excess abnormal returns of quintile portfolios of funds' subsamples sorted by coskewness (CS) and idiosyncratic skewness (IS) estimates. According to Titman and Tiu's (2011) hedge fund skill measure, we divide the hedge fund sample into high and low skilled funds. We estimate the Fung and Hsieh seven-factor model R-squared model via a rolling window of 36 monthly observations in each month and for each fund. A fund is classified as high (low) skilled if its R-squared is below (above) the median. Quintile portfolios are formed every month from January 1997 to December 2018 by sorting separately high and low skill funds based on their *CS* and *IS* values estimated via a rolling window of 36 monthly observations. Quintile 1 is the portfolio of funds with the lowest *CS/IS*, and Quintile 5 is the portfolio with the highest *CS/IS*. We report the equally–weighted returns for each quintile portfolio and a high – minus – low quintile portfolio and their generated alphas with respect to Fung and Hsieh, Carhart, and six-factor models, respectively. Newey-West adjusted *t*-statistics for the *5 – 1* portfolio returns are reported in parentheses. *,**,*** denote the statistical significance at 10%, 5% and 1%, respectively.

Panel A: *High Skilled Funds*	CS				IS			
Quintiles	Raw Returns	FH alpha	Carhart alpha	Six-factor alpha	Raw Returns	FH alpha	Carhart alpha	Six-factor alpha
1	0.610	0.422	0.364	0.447	0.320	0.213	0.1653	0.211
2	0.481	0.362	0.314	0.355	0.433	0.311	0.255	0.322
3	0.438	0.352	0.304	0.325	0.455	0.328	0.281	0.329
4	0.407	0.313	0.267	0.298	0.546	0.401	0.361	0.400
5	0.420	0.252	0.209	0.321	0.598	0.451	0.388	0.478
5-1	-0.189**	-0.170*	-0.155*	-0.125	0.277***	0.237***	0.223***	0.267***
	(-1.97)	(-1.78)	(-1.66)	(-1.27)	(4.70)	(3.88)	(3.34)	(3.41)

Panel B: *Low Skilled Funds*	CS				IS			
Quintiles	Raw Returns	FH alpha	Carhart alpha	Six-factor alpha	Raw Returns	FH alpha	Carhart alpha	Six-factor alpha
1	0.475	0.062	0.174	0.195	0.365	0.052	0.103	0.137
2	0.440	0.165	0.208	0.210	0.381	0.071	0.148	0.222
3	0.404	0.146	0.178	0.237	0.435	0.128	0.174	0.227
4	0.485	0.174	0.215	0.292	0.481	0.153	0.196	0.199
5	0.425	0.001	0.061	0.193	0.548	0.196	0.263	0.365
5-1	-0.031	-0.061	-0.112	-0.002	0.182*	0.143	0.159	0.228**
	(-0.25)	(-0.46)	(-0.82)	(-0.01)	(1.77)	(1.45)	(1.38)	(2.08)

Table 7. Skewness estimates and time-varying betas

This table reports the coefficient estimates from Fama – MacBeth (1973) cross-sectional regressions of the funds' estimated coskewness (CS) and idiosyncratic skewness (IS), of the *CS* and *IS* estimates of funds *not* due to time-varying betas based on public information (*NTVB*) and, due to time-variation in betas ($TVB = skewness\ estimate - NTVB$) associated with public information on funds excess one-month-ahead returns and on a constant. Following Ferson and Schadt (1996), the *NTVB* estimates are *CS* and *IS* of the excess return, including the alpha estimate and equation (11) residuals, respectively. CS, IS, and their *not* due to and time-varying betas estimates have been estimated via a rolling window of 36 monthly observations. Point estimates for funds are winsorized at 0.5% and 99.5%. We have run the Fama – MacBeth (1973) regressions for each month and the full sample period January 1994 – December 2018. Newey-West corrected t-statistics are presented in parentheses. Coefficients that are significant at 1%, 5% and 10% are denoted by ***, **, *, respectively.

Panel A: Coskewness	CS	CS_{NTV}	CS_{TV}
Returns	-0.125*	-0.208***	0.097**
	(-1.81)	(-3.15)	(1.97)
Constant	-4.163***	-3.780***	-0.355
	(-3.81)	(-5.38)	(-0.73)
Adj. R^2	0.016	0.015	0.022
Panel B: Idios. Skewness	IS	IS_{NTV}	IS_{TV}
Returns	0.003 ***	0.002***	0.001***
	(4.96)	(4.65)	(3.36)
Constant	-0.026***	-0.020***	-0.001***
	(-4.19)	(-3.48)	(-4.47)
Adj. R^2	0.006	0.005	0.002

Table 8. Fama – MacBeth regressions of matrix completed one-month ahead hedge fund returns on skewness estimates and control variables

This table reports the coefficient estimates from Fama – MacBeth (1973) cross-sectional regressions of one – month ahead funds excess returns on a constant, the funds' estimated coskewness (CS) and idiosyncratic skewness (IS), and a set of control variables (i.e., past month's return, incentive fee, redemption period, minimum investment and dummy for lockup). *Matrix completion* has been used to retrieve the full matrix of funds returns. Point estimates for funds are winsorized at 0.5% and 99.5%. We have run the Fama – MacBeth (1973) regressions for each month and the full sample period January 1994 – December 2018. Coskewness (CS) and idiosyncratic skewness (IS) have been estimated via a rolling window of 36 monthly observations. Newey-West corrected *t*-statistics are presented in parentheses. Coefficients that are significant at 1%, 5% and 10% are denoted by ***, **, *, respectively.

Variable	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
CS	-0.009 **		-0.009**	-0.007**		-0.007**
	(-2.70)		(-2.63)	(-2.52)		(-2.44)
IS		0.175 ***	0.184***		0.130***	0.137***
		(4.11)	(4.57)		(4.02)	(4.44)
Lret				0.130***	0.133***	0.126***
				(7.76)	(7.94)	(7.68)
InFee				0.000	-0.000	-0.000
				(0.15)	(-0.69)	(-0.44)
RedPer				0.000**	0.000***	0.000***
				(2.51)	(3.15)	(2.83)
MinInv				-0.000***	-0.000***	-0.000***
				(-5.92)	(-5.58)	(-5.55)
DumLo				0.061***	0.058***	0.057***
				(7.16)	(6.75)	(6.72)
Adj. R^2	0.02	0.005	0.03	0.10	0.09	0.10

Table 9. Fama – MacBeth regressions of matrix completed one-month ahead hedge fund returns on skewness estimates and volatility

This table reports the coefficient estimates from Fama – MacBeth (1973) cross-sectional regressions of one – month ahead funds excess returns on a constant, the funds' estimated coskewness (CS) and idiosyncratic skewness (IS), and a set of risk variables (i.e., total volatility, systematic and unsystematic risk). *Matrix completion* has been used to retrieve the full matrix of funds returns. Point estimates for funds are winsorized at 0.5% and 99.5%. We have run the Fama – MacBeth (1973) regressions for each month and the full sample period January 1994 – December 2018. Coskewness (CS), idiosyncratic skewness (IS), total volatility (Vol), systematic (SR), and unsystematic risk (USR) have been estimated via a rolling window of 36 monthly observations, according to Bali et al. (2012). Newey-West corrected *t*-statistics are presented in parentheses. Coefficients that are significant at 1%, 5% and 10% are denoted by ***, **, *, respectively.

Variable	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
CS	-0.007**		-0.006**	-0.006**		-0.006**
	(-2.11)		(-2.05)	(-2.03)		(-1.97)
IS		0.140 ***	0.151***		0.147***	0.154***
		(3.10)	(3.56)		(3.52)	(3.92)
Vol	0.011***	0.013***	0.011***			
	(4.37)	(4.55)	(4.25)			
SR				0.025***	0.028***	0.026***
				(2.88)	(3.13)	(2.92)
USR				-0.003	-0.003	-0.003
				(-0.55)	(-0.62)	(-0.67)
Adj. R^2	0.06	0.05	0.06	0.09	0.08	0.10

Table 10. Univariate portfolios sorts of matrix completed hedge funds by skewness estimates

This table presents the one-month ahead average excess abnormal returns of quintile portfolios of funds sorted by coskewness (CS) and idiosyncratic skewness (IS) estimates. *Matrix completion* has been used to retrieve the full matrix of funds returns. Quintile portfolios are formed every month from January 1997 to December 2018 by sorting funds based on their *CS* and *IS* values estimated via a rolling window of 36 monthly observations. Quintile 1 is the portfolio of funds with the lowest *CS/IS*, and Quintile 5 is the portfolio with the highest *CS/IS*. We report the equally–weighted returns for each quintile portfolio and a high – minus – low quintile portfolio (*5 – 1*) and their generated alphas with respect to Fung and Hsieh, Carhart, and six–factor models, respectively. Newey-West adjusted *t*-statistics for the *5 – 1* portfolio returns are reported in parentheses. *,**,*** denote the statistical significance at 10%, 5% and 1%, respectively.

	CS				IS			
Quintiles	Raw Returns	FH alpha	Carhart alpha	Six-factor alpha	Raw Returns	FH alpha	Carhart alpha	Six-factor alpha
1	0.469***	0.286***	0.267***	0.282***	0.220**	0.119*	0.077	0.075
	(3.13)	(3.38)	(3.46)	(3.66)	(2.17)	(1.74)	(1.31)	(1.25)
2	0.311***	0.207***	0.180***	0.191***	0.278***	0.150***	0.131***	0.135***
	(3.77)	(4.31)	(4.25)	(4.96)	(2.91)	(2.91)	(2.66)	(2.84)
3	0.231***	0.153***	0.126***	0.133***	0.288***	0.174***	0.145***	0.152***
	(3.57)	(3.98)	(3.63)	(4.01)	(3.17)	(3.44)	(3.20)	(3.51)
4	0.192***	0.112***	0.082**	0.081*	0.304***	0.175***	0.161***	0.170***
	(2.76)	(2.65)	(1.97)	(1.89)	(3.40)	(3.80)	(3.66)	(4.16)
5	0.286***	0.143**	0.114**	0.145**	0.400***	0.284***	0.258***	0.301***
	(2.73)	2.28	(1.99)	(2.36)	(4.47)	(5.43)	(5.38)	(6.62)
5-1	-0.183**	-0.142**	-0.153**	-0.138*	0.180***	0.165***	0.181***	0.226***
	(-2.48)	(-2.16)	(-2.29)	(-1.79)	(4.25)	(3.81)	(3.43)	(4.15)